Implications of Precision Electroweak Measurements for the Standard Model Higgs Boson[a]

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We summarize the status of the Standard Model with special emphasis on the extraction of the Higgs boson mass using Bayesian inference.

1 Introduction

Besides the recent high precision measurements of the W mass [1,2], M_W, the most important input into precision tests of electroweak theory continues to come from the Z factories LEP 1 [1] and SLC [3]. The vanguard of the physics program at LEP 1 is the analysis of the Z lineshape. Its parameters are the Z mass, M_Z, the total Z width, Γ_Z, the hadronic peak cross section, $\sigma_{\rm had}$, and the ratios of hadronic to leptonic decay widths, $R_\ell = \frac{\Gamma(\rm had)}{\Gamma(\ell^+\ell^-)}$, where $\ell = e$, μ, or τ. They are determined in a common fit with the leptonic forward-backward (FB) asymmetries, $A_{FB}(\ell) = \frac{3}{4} A_e A_\ell$. With f denoting the fermion index,

$$A_f = \frac{2 v_f a_f}{v_f^2 + a_f^2} \tag{1}$$

is defined in terms of the vector ($v_f = I_{3,f} - 2 Q_f \sin^2\theta_f^{\rm eff}$) and axial-vector ($a_f = I_{3,f}$) $Zf\bar{f}$ coupling; Q_f and $I_{3,f}$ are the electric charge and third component of isospin, respectively, and $\sin^2\theta_f^{\rm eff} \equiv \bar{s}_f^2$ is an effective mixing angle.

The polarization of the electron beam at the SLC allows for competitive and complementary measurements with a much smaller number of Z's than at LEP. In particular, the left-right (LR) cross section asymmetry, $A_{LR} = A_e$, represents the most precise determination of the weak mixing angle by a single experiment (SLD). [3] Mixed FB-LR asymmetries, $A_{LR}^{FB}(f) = \frac{3}{4} A_f$, single out the final state coupling of the Z boson.

For several years there has been an experimental discrepancy at the 2σ level between A_ℓ from LEP and the SLC. With the 1997/98 high statistics run

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at the SLC, and a revised value for the FB asymmetry of the τ polarization, $\mathcal{P}_\tau^{FB}$, the two determinations are now consistent with each other,

$$\begin{aligned} A_\ell(\text{LEP}) &= 0.1470 \pm 0.0027, \\ A_\ell(\text{SLD}) &= 0.1503 \pm 0.0023. \end{aligned} \tag{2}$$

The LEP value is from $A_{FB}(\ell)$, $\mathcal{P}_\tau$, and $\mathcal{P}_\tau^{FB}$, while the SLD value is from A_{LR} and $A_{LR}^{FB}(\ell)$. The data is consistent with lepton universality, which is assumed here. There remains a 2.5σ discrepancy between the two most precise determinations of $\bar{s}_\ell^2$, i.e. A_{LR} and $A_{FB}(b)$ (assuming no new physics in A_b).

Of particular interest are the results on the heavy flavor sector [1] including $R_q = \frac{\Gamma(q\bar{q})}{\Gamma(\text{had})}$, $A_{FB}(q)$, and $A_{LR}^{FB}(q)$, with $q = b$ or c. At present, there is some discrepancy in $A_{LR}^{FB}(b) = \frac{3}{4}A_b$ and $A_{FB}(b) = \frac{3}{4}A_e A_b$, both at the 2σ level. Using the average of Eqs. (2), $A_\ell = 0.1489 \pm 0.0018$, both can be interpreted as measurements of A_b. From $A_{FB}(b)$ one would obtain $A_b = 0.887 \pm 0.022$, and the combination with $A_{LR}^{FB}(b) = \frac{3}{4}(0.867 \pm 0.035)$ would yield $A_b = 0.881 \pm 0.019$, which is almost 3σ below the SM prediction. Alternatively, one could use $A_\ell(\text{LEP})$ above (which is closer to the SM prediction) to determine $A_b(\text{LEP}) = 0.898 \pm 0.025$, and $A_b = 0.888 \pm 0.020$ after combination with $A_{LR}^{FB}(b)$, i.e., still a 2.3σ discrepancy. An explanation of the 5–6% deviation in A_b in terms of new physics in loops, would need a 25–30% radiative correction to $\hat{\kappa}_b$, defined by $\bar{s}_b^2 \equiv \hat{\kappa}_b \sin^2 \hat{\theta}_{\overline{\text{MS}}}(M_Z)$. Only a new type of physics which couples at the tree level preferentially to the third generation [4], and which does not contradict R_b (including the off-peak measurements by DELPHI [5]), can conceivably account for a low A_b. Given this and that none of the observables deviates by 2σ or more, we can presently conclude that there is no compelling evidence for new physics in the precision observables, some of which are listed in Table 1.

2 Bayesian Higgs mass inference

The data show a strong preference for a low $M_H \sim \mathcal{O}(M_Z)$,

$$M_H = 107^{+67}_{-45} \text{ GeV}, \tag{3}$$

where the central value (of the global fit to all precision data, including m_t) maximizes the likelihood, $N e^{-\chi^2(M_H)/2}$. Correlations with other parameters, ξ^i, are accounted for, since minimization w.r.t. these is understood, $\chi^2 \equiv \chi^2_{\text{min}}$.

Bayesian methods, on the other hand, are based on Bayes theorem [6],

$$p(M_H|\text{data}) = \frac{p(\text{data}|M_H)p(M_H)}{p(\text{data})}, \tag{4}$$

Table 1: Principal precision observables from CERN, FNAL, SLAC, and elsewhere. Shown are the experimental results, the SM predictions, and the pulls. The SM errors are from the uncertainties in M_Z, $\ln M_H$, m_t, $\alpha(M_Z)$, and α_s. They have been treated as Gaussian and their correlations have been taken into account. $\bar{s}^2_\ell(Q_{FB}(q))$ is the weak mixing angle from the hadronic charge asymmetry; R^- and R^ν are cross section ratios from deep inelastic ν-hadron scattering; $g^{\nu e}_{V,A}$ are effective four-Fermi coefficients in ν-e scattering; and the Q_W are the weak charges from parity violation measurements in atoms. The uncertainty in the $b \to s\gamma$ observable includes theoretical errors from the physics model, the finite photon energy cut-off, and from uncalculated higher order effects. There are other precision observables which are not shown but included in the fits. Very good agreement with the SM is observed. Only A_{LR} and the two measurements sensitive to A_b discussed in the text, show some deviation, but even those are below 2σ.

Quantity	Group(s)	Value	Standard Model	pull
M_Z [GeV]	LEP	91.1867 ± 0.0021	91.1865 ± 0.0021	0.1
Γ_Z [GeV]	LEP	2.4939 ± 0.0024	2.4957 ± 0.0017	-0.8
$\sigma_{\rm had}$ [nb]	LEP	41.491 ± 0.058	41.473 ± 0.015	0.3
R_e	LEP	20.783 ± 0.052	20.748 ± 0.019	0.7
R_μ	LEP	20.789 ± 0.034	20.749 ± 0.019	1.2
R_τ	LEP	20.764 ± 0.045	20.794 ± 0.019	-0.7
$A_{FB}(e)$	LEP	0.0153 ± 0.0025	0.0161 ± 0.0003	-0.3
$A_{FB}(\mu)$	LEP	0.0164 ± 0.0013		0.2
$A_{FB}(\tau)$	LEP	0.0183 ± 0.0017		1.3
R_b	LEP + SLD	0.21656 ± 0.00074	0.2158 ± 0.0002	1.0
R_c	LEP + SLD	0.1735 ± 0.0044	0.1723 ± 0.0001	0.3
$A_{FB}(b)$	LEP	0.0990 ± 0.0021	0.1028 ± 0.0010	-1.8
$A_{FB}(c)$	LEP	0.0709 ± 0.0044	0.0734 ± 0.0008	-0.6
A_b	SLD	0.867 ± 0.035	0.9347 ± 0.0001	-1.9
A_c	SLD	0.647 ± 0.040	0.6676 ± 0.0006	-0.5
$A_{LR} + A_\ell$	SLD	0.1503 ± 0.0023	0.1466 ± 0.0015	1.6
$\mathcal{P}_\tau : A_e + A_\tau$	LEP	0.1452 ± 0.0034		-0.4
$\bar{s}^2_\ell(Q_{FB})$	LEP	0.2321 ± 0.0010	0.2316 ± 0.0002	0.5
m_t [GeV]	Tevatron	173.8 ± 5.0	171.4 ± 4.8	0.5
M_W [GeV]	all	80.388 ± 0.063	80.362 ± 0.023	0.4
R^-	NuTeV	$0.2277 \pm 0.0021 \pm 0.0007$	0.2297 ± 0.0003	-0.9
R^ν	CCFR	$0.5820 \pm 0.0027 \pm 0.0031$	0.5827 ± 0.0005	-0.2
R^ν	CDHS	$0.3096 \pm 0.0033 \pm 0.0028$	0.3089 ± 0.0003	0.2
R^ν	CHARM	$0.3021 \pm 0.0031 \pm 0.0026$		-1.7
$g_V^{\nu e}$	all	-0.041 ± 0.015	-0.0395 ± 0.0004	-0.1
$g_A^{\nu e}$	all	-0.507 ± 0.014	-0.5063 ± 0.0002	-0.1
$Q_W(\mathrm{Cs})$	Boulder	$-72.41 \pm 0.25 \pm 0.80$	-73.10 ± 0.04	0.8
$Q_W(\mathrm{Tl})$	all	$-114.8 \pm 1.2 \pm 3.4$	-116.7 ± 0.1	0.5
$\frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\nu)}$	CLEO	$3.26^{+0.75}_{-0.68} \times 10^{-3}$	$3.14^{+0.19}_{-0.18} \times 10^{-3}$	0.1

which must be satisfied once the *likelihood*, $p(\text{data}|M_H)$, and *prior* distribution, $p(M_H)$, are specified. $p(data) \equiv \int p(\text{data}|M_H)p(M_H)dM_H$ in the denominator provides for the proper normalization of the *posterior* distribution on the l.h.s. The prior can contain additional information not included in the likelihood model, or chosen to be *non-informative*.

Occasionally, the Bayesian method is criticized for the need of a prior, which would introduce unnecessary subjectivity into the analysis. Indeed, care and good judgement is needed, but the same is true for the likelihood model, which has to be specified in both approaches. Moreover, it is appreciated among Bayesian practitioners, that the explicit presence of the prior can be advantageous: it manifests model assumptions and allows for sensitivity checks. From the theorem (4) it is also clear that the maximum likelihood method corresponds, mathematically, to a particular choice of prior. Thus Bayesian methods differ rather in attitude: by their strong emphasis on the entire posterior distribution and by their first principles setup.

Given extra parameters, ξ^i, the distribution function of M_H is defined as the marginal distribution, $p(M_H|\text{data}) = \int p(M_H, \xi^i|\text{data}) \prod_i p(\xi^i)d\xi^i$. If the posterior factorizes, $p(M_H, \xi^i) = p(M_H)p(\xi^i)$, the ξ^i dependence can be ignored. If not, but $p(\xi^i|M_H)$ is (approximately) multivariate normal, then

$$\chi^2(M_H, \xi^i) = \chi^2_{\text{min}}(M_H) + \frac{1}{2}\frac{\partial^2\chi^2(M_H)}{\partial\xi_i\partial\xi_j}(\xi^i - \xi^i_{\text{min}}(M_H))(\xi^j - \xi^j_{\text{min}}(M_H)). \quad (5)$$

The latter applies to our case, where $\xi^i = (m_t, \alpha_s, \alpha(M_Z))$. Integration yields,

$$p(M_H|\text{data}) \sim \sqrt{\det E}\, e^{-\chi^2_{\text{min}}(M_H)/2}, \quad (6)$$

where the ξ^i error matrix, $E = (\frac{\partial^2\chi^2(M_H)}{\partial\xi_i\partial\xi_j})^{-1}$, introduces a correction factor with a mild M_H dependence. It corresponds to a shift relative to the standard likelihood model, $\chi^2(M_H) = \chi^2_{\text{min}}(M_H) + \Delta\chi^2(M_H)$, where

$$\Delta\chi^2(M_H) \equiv \ln\frac{\det E(M_H)}{\det E(M_Z)}. \quad (7)$$

For example, $\Delta\chi^2(300 \text{ GeV}) \sim 0.1$, which would *tighten* the M_H upper limit by at most a few GeV. At present, we neglect this effect.

We choose $p(M_H)$ as the product of M_H^{-1}, corresponding to a uniform (non-informative) distribution in $\log M_H$, times the exclusion curve from LEP 2. [7] This curve is from Higgs searches at center of mass energies up to 183 GeV. We find the 90 (95, 99)% confidence upper limits,

$$M_H < 220 \ (255,\ 335) \text{ GeV}. \quad (8)$$

Theory uncertainties from uncalculated higher orders increase the 95% CL by about 5 GeV. These limits are robust within the SM, but we caution that the results on M_H are strongly correlated with certain new physics parameters [8].

The one-sided confidence interval (8) is not an exclusion limit. For example, the 95% upper limit of the standard uniform distribution, $x \in [0,1]$, is at $x = 0.95$, but all values of x are equally likely, and $x > 0.95$ cannot be excluded. If there is a discrete set of competing hypotheses, H_i, one can use Bayes factors, $p(\text{data}|H_i)/p(\text{data}|H_j)$, for comparison. For example, LEP 2 rejects a standard Higgs boson with $M_H < 90$ GeV at the 95% CL, because

$$\frac{p(\text{data}|M_H = M_0)}{p(\text{data}|M_H \neq M_0)} < 0.05 \qquad \forall\, M_0 < 90 \text{ GeV}. \tag{9}$$

On the other hand, the probability for $M_H < 90$ GeV is only 5×10^{-4}.

One could similarly note, that $p(M_H = M_0) < 0.05\, p(M_H = 107 \text{ GeV})$ for $M_0 > 334$ GeV; but the (arbitrary) choice of the best fit M_H value as reference hypothesis is hardly justifiable. This affirms that variables continuously connecting a set of hypotheses should be treated in a fully Bayesian analysis.

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